

33.75. Visualize: Please refer to Figure P33.75.

Solve: (a) After a long time has passed the current will no longer be changing. With steady currents, the potential difference across the inductor is $\Delta V_L = -L(dI/dt) = 0$ V. An ideal inductor has no resistance ($R = 0 \Omega$), so the inductor simply acts like a wire. The circuit is simply that of a battery and resistor R , so the current is $I_0 = \Delta V_{\text{bat}}/R$.

(b) In general, we need to apply Kirchhoff's loop law to the circuit. Starting with the battery and going clockwise, the loop law is

$$\Delta V_{\text{bat}} + \Delta V_R + \Delta V_L = \Delta V_{\text{bat}} - IR - L \frac{dI}{dt} = 0$$
$$\frac{dI}{dt} = \frac{\Delta V_{\text{bat}}}{L} - \frac{IR}{L} = \frac{R}{L} \left(\frac{\Delta V}{R} - I \right) = \frac{R}{L} (I_0 - I)$$

This is a differential equation that we can solve by direct integration. The current is $I = 0$ A at $t = 0$ s, so separate the current and time variables and then integrate from 0 A at 0 s to current I at time t :

$$\int_0^t \frac{dI}{I_0 - I} = \frac{R}{L} \int_0^t dt \Rightarrow -\ln(I_0 - I)|_0^t = \frac{R}{L} t|_0^t \Rightarrow -\ln\left(\frac{I_0 - I}{I_0}\right) = \frac{t}{L/R}$$

Taking the exponential of both sides gives

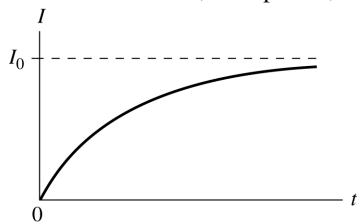
$$\frac{I_0 - I}{I_0} = e^{-t/(L/R)} \Rightarrow I_0 - I = I_0 e^{-t/(L/R)}$$

Finally, solving for I gives

$$I = I_0 (1 - e^{-t/(L/R)})$$

The current is 0 A at $t = 0$ s, as expected, and exponentially approaches I_0 as $t \rightarrow \infty$.

(c)



Assess: The current is zero at the start and approaches the steady final value. The behavior is similar to the charging of a capacitor.